Melville Senior High School

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 3&4

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures | | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
   Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Markers use only		
Question	Maximum	Mark
1	6	
2	6	
3	8	
4	7	
5	5	
6	5	
7	6	
8	9	
S1 Total	52	
S1 Wt (×0.6731)	35%	
S2 Wt	65%	
Total	100%	

See next page

Section One: Calculator-free **35% (52 Marks)**

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(6 marks)**

The function f is defined by

$$f(x) = \frac{6}{2x + x^3} = \frac{ax}{x^2 + 2} + \frac{b}{x}.$$

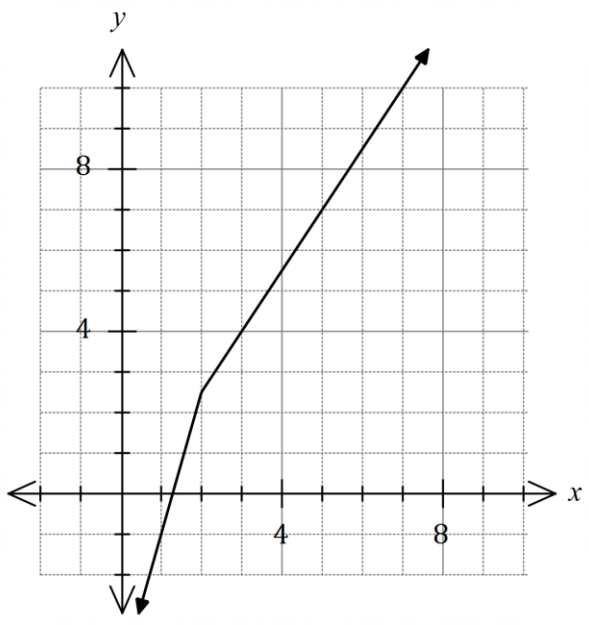
(a) Determine the value of the constant a and the value of the constant b. (3 marks)

(b) Hence, or otherwise, determine the value of $\int_1^2 f(x)\,dx$ in simplest form. (3 marks)

See next page

Question 2 **(6 marks)**

The function f is defined by $f(x) = 2.5(x-1) - |x-2|$. The graph of $y = f(x)$ is shown below.



(a) State the value of $f^{-1}(7)$. (1 mark)

(b) Sketch the graph of $y = f^{-1}(x)$ on the axes above. (2 marks)

(c) Solve $f(x) = f^{-1}(x)$. (3 marks)

Question 3 **(8 marks)**

The function f is defined by $f(x) = x^2 - 4x + 7,\ x \geq 0$.

(a) Determine the range of f. (3 marks)

The function g is defined by $g(x) = 8 - \sqrt{x+6},\ x \geq -6$.

(b) Determine an expression for $g \circ f(x)$. (2 marks)

(c) Determine the domain and range of $g \circ f(x)$. (3 marks)

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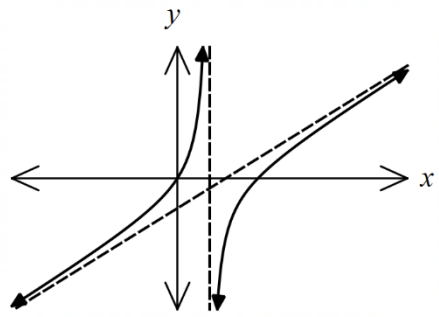
Question 4 **(7 marks)**

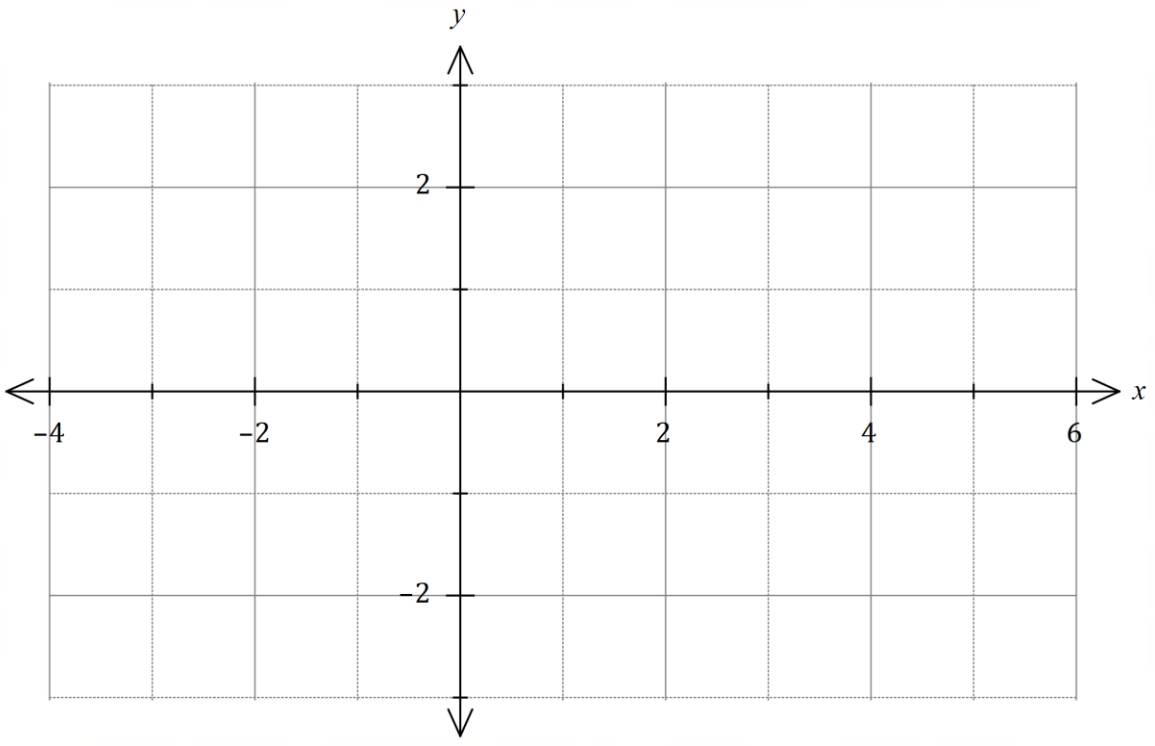
Let $f(x) = \dfrac{x(2x-5)}{x-1}$.

The graph of $y = f(x)$ is shown at right.



(a) Determine the equation of each asymptote shown on the graph of $y = f(x)$. (3 marks)

(b) On the axes below, sketch the graph of $y = \dfrac{x-1}{x(2x-5)}$. (4 marks)



Question 5 **(5 marks)**

Use the substitution $x = \sqrt{3}\tan\theta$ to evaluate $\int_0^1 \frac{2}{x^2+3}\,dx$

See next page

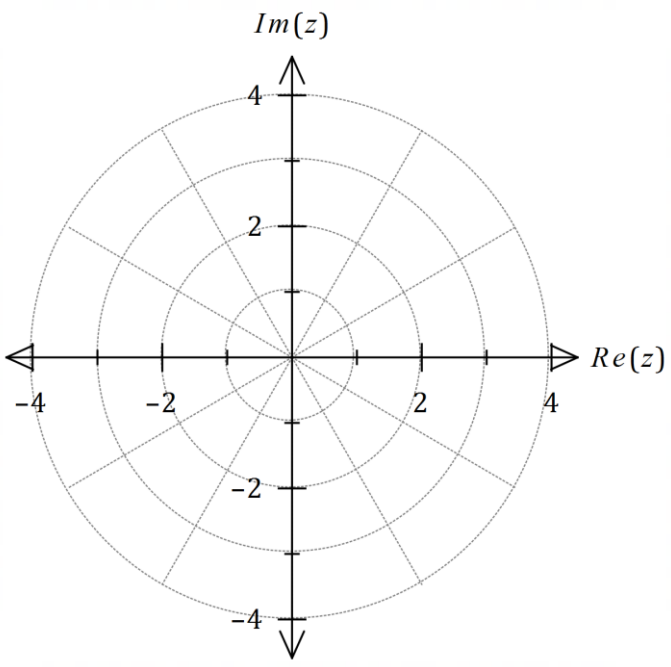
Question 6 **(5 marks)**

Consider the equation $z^3 - 5z^2 + 15z - 18 = 0,\ z \in \mathbb{C}$.

One root of the equation is $z = 3\,\text{cis}\left(\frac{\pi}{3}\right)$.

(a) Solve the equation, giving all solutions in Cartesian form. (4 marks)

(b) Locate all the roots of the equation on the Argand diagram below. (1 mark)



Question 7 **(6 marks)**

The equations of three planes are $x + ay + bz = 1$, $2x - 3y + z = 5$ and $2x - y + 3z = -1$, where a and b are integer constants.

Elimination can be used to reduce the system of equations to

$$\begin{aligned} x + ay + bz &= 1 \\ (2a + 3)y + (2b - 1)z &= -3 \\ (b - a - 2)z &= 3a + 3 \end{aligned}$$

(a) Determine any necessary restrictions on the value of a and/or the value of b for the system of equations to have

(i) a unique solution. (1 mark)

(ii) no solutions. (1 mark)

(b) For a particular value of a and value of b, the three planes intersect in a straight line. Determine the vector equation of this line. (4 marks)

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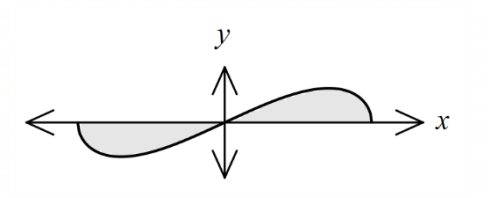
Question 8 **(9 marks)**

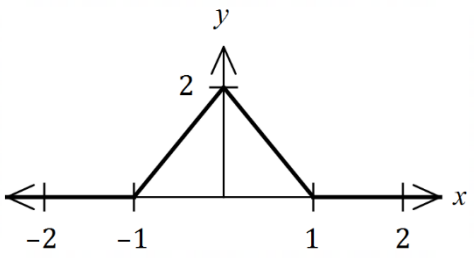
(a) The graph of the curve $y = 6x\sqrt{4 - x^2}$ is shown at right.



Using the substitution $u = 4 - x^2$, or otherwise, determine the area between the curve and the x-axis.

(6 marks)

(b) The graph of $y = f(x)$ is shown at right.



Determine the value of $\int_{-1}^{1} f(1 - x^2)\, dx$.

(3 marks)

End of questions

Supplementary page

Question number: _____